Two-Dimensional Radiation in a Cylinder with Spatially Varying Albedo

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Introduction

IN recent years, interest in multidimensional radiative heat transfer has increased mainly because of its numerous practical applications in combustion chambers, furnaces, and other high-temperature devices. In such systems, assumptions of constant properties and isotropic scattering are not realistic. Only a limited amount of work is available in the literature on the solution of radiation problems involving anisotropic scattering and spatially varying radiative properties.[1] In this paper, we use the *discrete ordinates method* or the *S_N method*[2-4] to develop a general, accurate scheme for computing two-dimensional radiative transfer in an axisymmetric cylindrical enclosure containing an absorbing, emitting, and anisotropically scattering medium, with spatially varying albedo.

Analysis

Equation of Radiative Transfer

For an absorbing, emitting, and anisotropically scattering gray medium in local thermodynamic equilibrium, the equation of radiative transfer can be written as[5]

$$(\mathbf{\Omega}\cdot\nabla^*)I(\tau,\mathbf{\Omega}) + I(\tau,\mathbf{\Omega}) = (1-\omega)I_b(T) + \frac{\omega}{4\pi}\int_{\Omega'=4\pi} p(\mathbf{\Omega}'\cdot\mathbf{\Omega})I(\tau,\mathbf{\Omega}')\mathrm{d}\Omega' \tag{1a}$$

where $I(\tau, \mathbf{\Omega})$ is the radiation intensity at location τ in the direction $\mathbf{\Omega}$, ω is the single scattering albedo, $I_b(T)$ is the blackbody radiation intensity, ∇^* is dimensionless gradient operator, and $p(\mathbf{\Omega}'\cdot\mathbf{\Omega})$ is the phase function from the incoming direction $\mathbf{\Omega}'$ to the outgoing direction $\mathbf{\Omega}$. The phase function $p(\mathbf{\Omega}'\cdot\mathbf{\Omega})$ can be epxressed in terms of the Legendre polynomials, $P_n(\nu)$, as

$$p(\mathbf{\Omega}'\cdot\mathbf{\Omega}) = p(\nu) = \sum_{n=0}^{N^*} a_n P_n(\nu) \tag{1b}$$

If the surface bounding the medium is gray, diffuse emitter and diffuse reflector, then the boundary condition is given by

$$I(s,\mathbf{\Omega}) = \varepsilon I_b(s) + \frac{\rho}{\pi}\int_{\mathbf{n}\cdot\Omega\Omega\Omega\Omega'>0} (\mathbf{n}\cdot\mathbf{\Omega}')I(s,\mathbf{\Omega}')\mathrm{d}\Omega' \qquad \mathbf{n}\cdot\mathbf{\Omega} < 0 \tag{2}$$

where $I(s, \mathbf{\Omega})$ is the boundary intensity at s, ε and ρ are the emissivity and reflectivity, $\mathbf{n}$ is the outward drawn unit normal vector.

Discrete Ordinates Form

Figure 1 shows the coordinates and the nomenclature. Equation of radiative transfer, i.e., Eq. (1a), is given in the discrete ordinates form as[2]

$$\frac{\mu_m}{\tau_r}\frac{\partial(\tau_r I_m)}{\partial\tau_r} - \frac{1}{\tau_r}\frac{\partial(\eta_m I_m)}{\partial\phi} + \xi_m\frac{\partial I_m}{\partial\tau_z} + I_m = (1-\omega)I_b + \frac{\omega}{4\pi}\sum_{m'} w_{m'}p_{m'm}I_{m'} \tag{3a}$$

where subscript m and m' represent the discrete ordinates, w_m is the quadrature weight, and the phase function $p_{m'm}$ for general anisotropic scattering is given by

$$p_{m'm} = \sum_{n=0}^{N^*} a_n P_n(\nu_{m'm}) \quad \text{with} \quad \nu_{m'm} = \mu_{m'}\mu_m + \eta_{m'}\eta_m + \xi_{m'}\xi_m \tag{3b,c}$$

and $m = 1, 2, 3, \ldots\ldots, M$, where M is the total number of discrete ordinates. Boundary conditions given by Eq. (2) are expressed as

$$I_m = \varepsilon_1 I_{b1} + \frac{\rho_1}{\pi}\sum_{m'} w_{m'}\mu_{m'}I_{m'} \qquad \mu_{m'} > 0,\ \mu_m < 0,\ \tau_r = R \tag{4}$$

$$I_m = I_{m'} \qquad \mu_{m'} < 0,\ \mu_m > 0,\ \tau_r = 0 \tag{5}$$

$$I_m = \varepsilon_2 I_{b2} + \frac{\rho_2}{\pi}\sum_{m'} w_{m'}|\xi_{m'}|I_{m'} \qquad \xi_{m'} < 0,\ \xi_m > 0,\ \tau_z = -L \tag{6}$$

$$I_m = \varepsilon_3 I_{b3} + \frac{\rho_3}{\pi}\sum_{m'} w_{m'}\xi_{m'}I_{m'} \qquad \xi_{m'} > 0,\ \xi_m < 0,\ \tau_z = L \tag{7}$$

where R is the optical radius, $2L$ is the optical axis length, and the subscripts 1, 2, and 3 refer to the surfaces $\tau_r = R$, $\tau_z = -L$, and $\tau_z = L$, respectively.

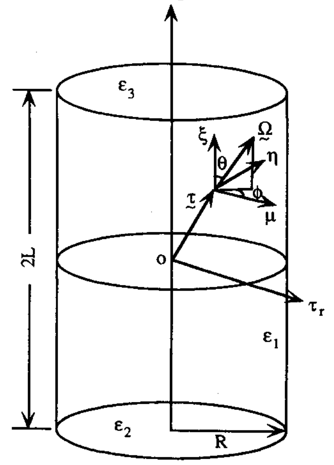

Fig. 1 Schematic of the physical system and coordinates.

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Solution of Discrete Ordinates Equations

A control volume form of the discrete ordinates equations is obtained by expressing the ϕ derivative term in Eq. (3a) in a difference form[3] and then integrating the resulting equation over a single mesh cell of volume $V = \pi[(\tau_r^{i+1})^2 - (\tau_r^i)^2]\Delta\tau_z^j$, where the areas of the cell surfaces are respectively, $A_i = 2\pi\tau_r^i\Delta\tau_z^j$, $A_{i+1} = 2\pi\tau_r^{i+1}\,\Delta\tau_z^j$, and $B_j = B_{j+1} = \pi[(\tau_r^{i+1})^2 - (\tau_r^i)^2]$. Multiplying the resulting equation by $2\pi\tau_r\, d\tau_r\, d\tau_z$ and integrating over the cell volume V gives

$$\mu_m(A_{i+1}I_m^{i+1} - A_iI_m^i) - (A_{i+1} - A_i)\left[\frac{\alpha_{m+1/2}I_{m+1/2} - \alpha_{m-1/2}I_{m-1/2}}{w_m}\right] + \xi_m(B_{j+1}I_m^{j+1} - B_jI_m^j) + VI_m = VS_m^* \tag{8a}$$

where

$$S_m^* = (1-\omega)I_b + \frac{\omega}{4\pi}\sum_{m'=1}^{M} w_{m'}p_{m'm}I_{m'} \tag{8b}$$

Here I_m, $I_{m-1/2}$, $I_{m+1/2}$, and S_m^* are evaluated at the center of cell V, and I_m^i, I_m^j, I_m^{i+1}, I_m^{j+1} are the intensities at the surfaces i, j, $i+1$, and $j+1$, respectively, of cell V. If the intensity is assumed to vary linearly with all the independent variables over the whole cell, then we have

$$I_m^i + I_m^{i+1} = I_m^j + I_m^{j+1} = I_{m+1/2} + I_{m-1/2} = 2I_m \tag{9}$$

which is called the *diamond difference scheme*. Equations (4–9) are solved using the procedure described in Ref. 2 to calculate the radiation intensity. The moment-matching technique[3] is applied to calculate the quadrature points and weights. The quadrature points for the S_4 scheme can be found in Ref. 4. Knowing the radiation intensity I_m, the net radiation fluxes $q_r(\boldsymbol{\tau})$ and $q_z(\boldsymbol{\tau})$ in the τ_r and τ_z directions, respectively, are determined from

$$q_r(\boldsymbol{\tau}) = \sum_m w_m\mu_mI_m \quad \text{and} \quad q_z(\boldsymbol{\tau}) = \sum_m w_m\xi_mI_m \tag{10a,b}$$

Results and Discussion

In order to test the accuracy of the present scheme, the following case is considered first. The medium is at a uniform temperature such that $I_b[T(\tau_r, \tau_z)] = 1$, while the boundaries are cold, i.e., $I_{bi} = 0$, $i = 1, 2, 3$. The medium absorbs, emits, and scatters isotropically. Figure 2 shows the radiation heat flux at the cylindrical surface plotted against the axial position for different values of the optical thickness $R = L$. The predictions of the discrete ordinates method are in good agreement with Thynell's results.[6]

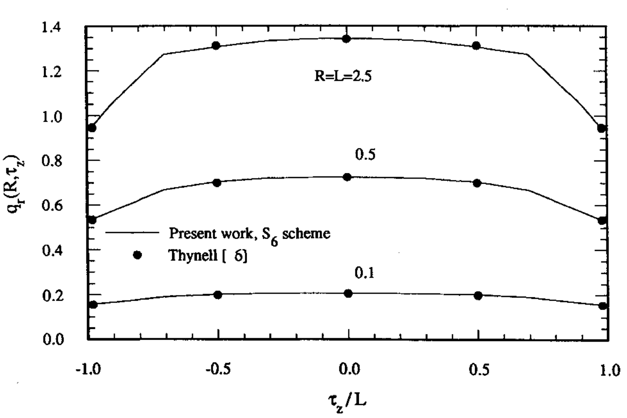

Fig. 2 The effects of the optical thickness on the heat flux for a hot medium enclosed by cold walls, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.5$, $\omega = 0.5$.

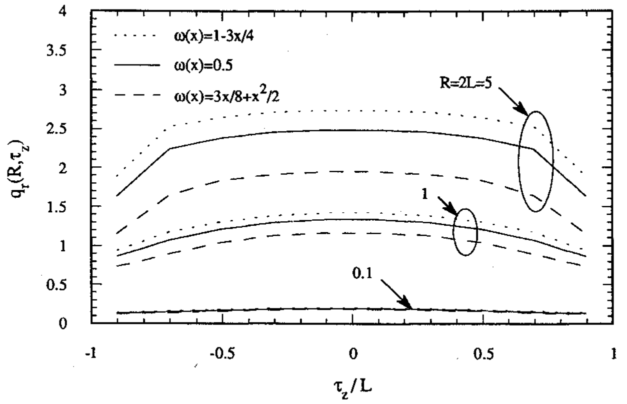

Fig. 3 The effects of the spatially varying albedo on the heat flux for a hot medium enclosed by cold walls, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$, $x = \tau_r/R$.

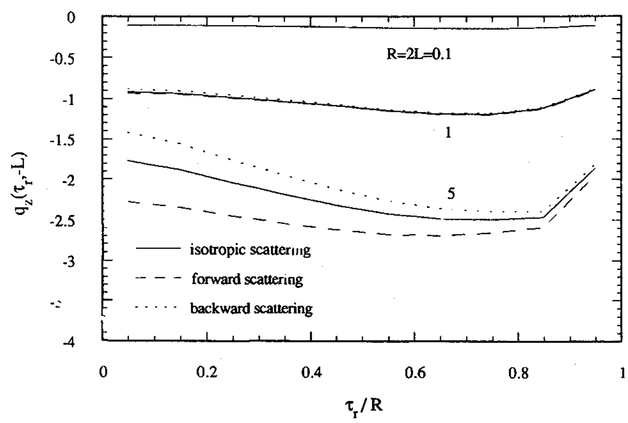

Fig. 4 The effects of the phase function on the heat flux for a hot medium enclosed by cold walls, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$, $\omega(x) = 1 - 3x/4$, $x = \tau_r/R$.

In most engineering problems, when the radiation properties of the medium vary with positions, average values of the properties are generally used for radiation calculations. Such an assumption can introduce errors. To illustrate the effects of the spatial variation of ω and the errors involved as a result of using the average radiation properties, we have chosen a linear and a quadratic variation of albedo with an average value of 0.5. Figure 3 shows the radiation flux at the cylindrical surface plotted against the axial position for different values of the optical thickness $R = 2L$. Note that for the optically thin ($R = 2L = 0.1$) case, the use of an average value $\omega = 0.5$ introduces only a small error, but for the case of larger optical thickness, i.e., $R = 2L = 5$, significant error is introduced in the wall heat flux.

To show the effects of the anisotropic scattering, two different scattering laws, the forward and the backward scattering, are considered by taking the coefficients a_n of Eq. (1b) the same as those in Ref. 1. Figure 4 shows the effects of forward and backward scattering on the wall heat flux compared with those of the isotropic scattering for different optical thicknesses plotted against the radial position.

Conclusion

The discrete ordinates method has been used to solve radiative heat transfer in an axisymmetric cylinder in order to examine the effects of spatially varying albedo and anisotropic scattering. The spatial variation of albedo has significant effects on the heat flux for the optically thick case, but for the optically thin case the effects are negligible.

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Determination of Optical Properties by Two-Dimensional Scattering

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Nomenclature

a	= absorption coefficient
D	= thickness of absorbing/scattering sample
$\mathbf{ds_1g}$	= source-to-volume exchange factor vector
ds_1g_i	= exchange factor between source and volume zone V_i
$\overline{gg}$	= volume-volume exchange factor matrix
g_ig_j	= exchange factor between volume zones V_i and V_j
I_0	= flux density of source
K	= extinction coefficient
L	= half-width of absorbing/scattering sample
Q_2	= heat transfer to detector
S	= error sum defined by Eq. (12)
$S_n(x)$	= function defined in Ref. 5
$\mathbf{s_2g}$	= detector-to-volume exchange factor vector
$(\mathbf{s_2g})^T$	= transpose of $\mathbf{s_2g}$
s_2g_i	= exchange factor between $2W$ and V_i
t_{ij}	= distance parameter defined by Eq. (9)
$\bar{V}$	= volume matrix
V_i	= volume zonal element
W	= half-width of detector window opening
W_{gi}	= radiosity of volume zone V_i
$2W$	= detector width
x_i	= coordinate of V_i or A_i
x_2	= coordinate measured along detector $2W$
z_i	= coordinate of V_i or A_i
τ	= transmissivity
ω	= scattering albedo

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I. Introduction

THE importance of radiative heat transfer in the assessment of insulation materials for aerospace applications is well known. For practical design and performance evaluation of many engineering components, the optical properties of insulation materials are important parameters that must be accurately determined.

Theoretically, the optical properties of material are related to its index of refraction and various geometric parameters (particle/fiber or void size distribution and number density) and can, in principle, be computed by Mie Theory. A number of such computations have been reported.[1,2] But due to the variation in the index of refraction and the geometric complexity, such calculations are quite tedious and have uncertain accuracy. Indeed, the most common approach currently utilized by the aerospace industry is to determine optical properties of materials empirically by in situ optical transmission measurement.

Fundamentally, the basis of the optical transmission measurement is the Beer-Lambert law. For a slab of thickness D, the transmissivity τ is related to the absorption coefficient a by the relation

$$\tau = e^{-aD} \tag{1}$$

Assuming that a remains constant within the material, repeated measurements of τ at different D provide a set of statistically reliable data from which a can be determined. Experiments performed at different wavelengths and slab temperatures will determine the spectral and temperature dependence of a.

But while the optical transmission measurement is quite effective in determining a for nonscattering materials, it is not applicable for scattering materials because of the multidimensional effect of scattering. For example, consider the transmission of a collimated line source through a two-dimensional planar slab as shown in Fig. 1. Data obtained by Lockheed for an aerospace insulation material, LI900, are shown in Fig. 2. It is apparent that the Beer-Lambert law is inadequate as it fails to account for the scattering contribution to τ. For a medium with extinction coefficient K and scattering albedo ω, τ is a function of the optical depth KD, the optical width KL, ω, and the optical width of the detecting area KW. Until now, the lack of a reliable functional relation between τ, KD, KL, KW, and ω is probably the primary factor leading to the existing large uncertainty in optical properties for most insulating scattering materials.

The objective of the present work is to show that based on a recently developed network formulation,[3] the relation between τ and the optical properties of a two-dimensional scattering material can be determined numerically with little complexity. Based on these results, two-dimensional transmission data are inverted to determine "best-fit" values for the optical properties. For simplicity, this procedure is illustrated only for an isotropically scattering material and a two-dimensional

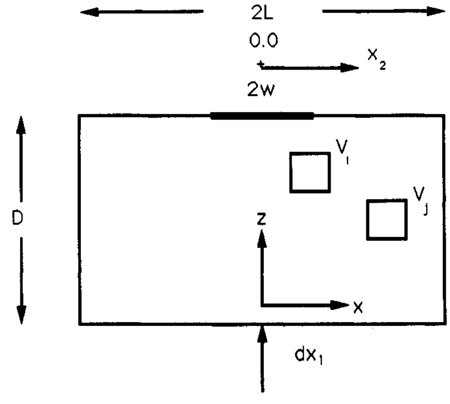

Fig. 1 Geometry of the two-dimensional planar scattering system.